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A NUMERICAL SOLUTION FOR PRECESSION AND NUTATION OF THE RIGID EARTH

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A numerical solution for the luni-solar precession and nutation of the rigid Earth is obtained and compared with the result from the analytical theories which are the basis of the current IAU precession and nutation formulae. We follow a simplified scheme of numerical calculation by modifying the equations of motion and thus avoiding a small step numerical integration. Some errors are found in the long periodic region of nutation in the current IAU theory.

Keywords: Precession and nutation, numerical integration, astronomical ephemeris.

1. Introduction

The values of precession in astronomical ephemeris are fundamentally based on the theories by Newcomb (1894, 1906) and Andoyer (1911). As for nutation, the authority is the 1980 IAU nutation theory (IAU, 1982) which was developed by Wahr (1981) for a non-rigid Earth using as the basis the nutation theory of the rigid Earth which was obtained by Kinoshita et al. (1979). This theory for rigid Earth is a thorough recomputation of the preceding work by Woolard (1953).

All these theories on precession and nutation for rigid Earth are analytical. The precision of the theory of precession is believed to be better than 0.0001″ except for the obliquity of ecliptic at the epoch and the coefficient of the linear term in the precession in longitude which have to be determined by observation. The nutation series for rigid Earth which is the basis of the 1980 IAU theory contains all the terms greater than 0.00005″, thus the precision being considered to be better than a few numbers at the place of 0.0001″.

Rather curiously, no numerical treatment has been attempted for precession and nutation. One of the reasons may be a great rapidity of the rotational motion of the Earth, i.e. one rotation in a day, which makes one feel at the first glance that the step in numerical integration of the equations of motion must be very small. Of course, another reason may be full confidence in the analytical theories.

Having a slight doubt about the precision of the current theories and introducing a method which enables to avoid numerical integration with a very small step, the present authors develop a numerical solution to luni-solar precession and nutation.

Because of some reasons a large computer, especially, precise ephemerides of the Moon and the Sun were not available in this study. Therefore the present work is in the nature of a pilot study and a more complete treatment should be made later.

2. Equations of motion

We describe the equations of motion for the rotation of rigid Earth in a fixed coordinate system.

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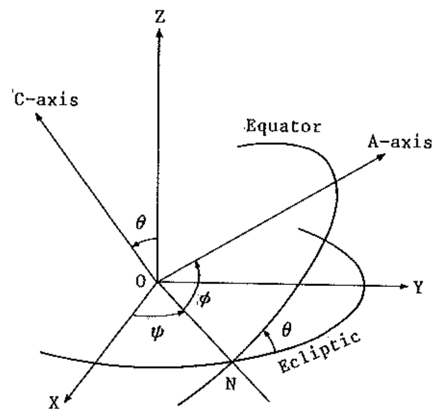

Figure 1. Eulerian angles.

The ecliptic and mean equinox of J2000.0 are adopted as this fundamental reference frame. The precession thus obtained can be compared directly with the expressions given by Lieske et al. (1977), but the result for the nutation must be reduced to the ecliptic and mean equinox of date before comparison because the nutation in astronomical ephemeris is referred to this frame.

Eulerian angles are used as the variables. They are the angles shown in Figure 1. The obliquity of ecliptic ϵ used in precession and nutation theory is equal to θ in the figure, although ϵ is usually described as the angle measured from the equator to the ecliptic at the ascending node of the ecliptic on the equator. While, it should be noticed that the angle expressed by the notation ψ in precession and nutation theory is measured on the ecliptic westward from the X-axis to the above mentioned node. Hence it is equal to $180^\circ - \psi$, ψ being one of the Eulerian angles.

In order to formulate the equations of motion, we first write the Lagrangean of the Earth rotating around its center of mass under external forces. It is given by

$$L = \frac{A}{2}(\dot{\theta}^2 + \dot{\psi}^2 \sin^2\theta) + \frac{C}{2}(\dot{\varphi} + \dot{\psi}\cos\theta)^2 + U(\psi, \theta; t). \tag{1}$$

In this equation, A and C are the moments of inertia with respect to an axis in the equatorial plane and the axis perpendicular to the plane, respectively, the latter of which we call the figure axis hereafter. We consider the Earth axially symmetrical so that $A = B$. U is the perturbing function due to the Moon and the Sun.

One of the Lagrangean equations obtained from (1) is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = 0.$$

From this we imediately get

$$\frac{d}{dt}(\dot{\varphi} + \dot{\psi}\cos\theta) = 0. \tag{2}$$

Hence,

$$\dot{\varphi} + \dot{\psi}\cos\theta = \omega \text{ (= const.)}, \tag{3}$$

where ω is the sidereal angular velocity of the rotation of the Earth. From this equation we can get φ if we have solved ψ and θ. Equation (2) is also written as

$$\ddot{\varphi} + \ddot{\psi} \cos\theta - \dot{\psi}\dot{\theta} \sin\theta = 0. \tag{4}$$

A second equation lead from (1) is

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0,$$

which results

$$A\ddot{\theta} - (A - C)\, \dot{\psi}^2 \sin\theta \cos\theta + C\,\dot{\psi}\,\dot{\varphi} \sin\theta - \frac{\partial U}{\partial \theta} = 0. \tag{5}$$

Also, from the Lagrangean (1) we have the equation:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\psi}} \right) - \frac{\partial L}{\partial \psi} = 0,$$

which gives

$$A\,(\ddot{\psi} \sin^2\theta + 2\,\dot{\psi}\,\dot{\theta} \sin\theta \cos\theta) + C\,(\ddot{\varphi} \cos\theta + \ddot{\psi} \cos^2\theta - \dot{\psi}\,\dot{\theta} \sin\theta \cos\theta)$$

$$- C\dot{\theta} \sin\theta\,(\dot{\varphi} + \dot{\psi} \cos\theta) - \frac{\partial U}{\partial \psi} = 0. \tag{6}$$

From equations (3), (4), (5) and (6) we obtain the following equations of motion for the orientation of the figure axis:

$$\ddot{\theta} = -\frac{C}{A}\,\omega \sin\theta\, \dot{\psi} + \sin\theta \cos\theta\, \dot{\psi}^2 + \frac{1}{A}\frac{\partial U}{\partial \theta},$$

$$\ddot{\psi} = \frac{C}{A}\,\omega \frac{1}{\sin\theta}\,\dot{\theta} - 2\,\frac{\cos\theta}{\sin\theta}\,\dot{\psi}\,\dot{\theta} + \frac{1}{A \sin^2\theta}\,\frac{\partial U}{\partial \psi}. \tag{7}$$

Fundamentally, all we have to do is to solve equations (7) by a numerical integration. This approach, however, is not practical. The motion of the figure axis contains the well-known Eulerian motion or free nutation which is a circular oscillation with the period of about one day in space.

This free nutation is independent of precession and nutation which are a forced motion, therefore it is not taken into account in the computation of precession and nutation. Nevertheless, we would have to solve this motion simultaneously in order to get the forced motion of the figure axis by performing a numerical integration, in which the step would have to be taken very small because of the rapidity of the motion.

3. Modification of the equations of motion

We now introduce a modification of the equations of motion (7). First, since the second terms in the right-hand members of the both equations are of a magnitude of $\dot{\psi}/\omega$ or 10^{-7} as compared to the first terms, we let them included in the first terms. In doing this we give $\dot{\psi}$ its average value $\bar{\dot{\psi}}$.

Next, we put

$$\frac{C}{A}\,\omega \sin\theta - \bar{\dot{\psi}} \sin\theta \cos\theta = P,$$

$$\frac{C}{A}\,\omega \frac{1}{\sin\theta} - 2\,\bar{\dot{\psi}}\,\frac{\cos\theta}{\sin\theta} = Q,$$

$$\frac{1}{A}\,\frac{\partial U}{\partial \theta} = f(\theta, \psi; t),$$

$$\frac{1}{A \sin^2 \theta} \frac{\partial U}{\partial \psi} = g(\theta, \psi; t). \tag{8}$$

Then we have

$$\ddot{\theta} = -P\,\dot{\psi} + f(\theta, \psi; t),$$
$$\ddot{\psi} = Q\,\dot{\theta} + g(\theta, \psi; t). \tag{9}$$

Now we consider P and Q to be constant and $f(\theta, \psi; t)$ and $g(\theta, \psi; t)$ functions of t alone. This approximation may be valid enough though we will not give a mathematically rigorous argument here.

Under this assumption, if there were not $f(t)$ and $g(t)$ in equations (9), they would have the following solution:

$$\dot{\theta} = \alpha \sin(\sqrt{QP}\, t + \gamma),$$
$$\dot{\psi} = -\frac{\sqrt{PQ}}{P} \alpha \cos(\sqrt{PQ}\, t + \gamma), \tag{10}$$

α and γ being arbitrary constants.

Guided by these expressions, we assume the following solution of equations (9):

$$\dot{\theta} = \alpha \sin(\sqrt{PQ} + \gamma) - \frac{1}{Q} g(t) + p(t),$$
$$\dot{\psi} = -\frac{\sqrt{PQ}}{P} \alpha \cos(\sqrt{PQ}\, t + \gamma) + \frac{1}{P} f(t) + q(t), \tag{11}$$

where $p(t)$ and $q(t)$ are functions of t.

Substitution of (11) into (9) gives

$$\dot{p}(t) = -P\,q(t) + \frac{1}{Q} \dot{g}(t),$$
$$\dot{q}(t) = Q\,p(t) - \frac{1}{P} \dot{f}(t).$$

By repeating the same procedure, we obtain as the solution of equations (9),

$$\dot{\theta} = \alpha \sin(\sqrt{PQ}\, t + \gamma) - \frac{1}{Q} g(t) + \frac{1}{PQ} \dot{f}(t) + \frac{1}{PQ^2} \ddot{g}(t) - \frac{1}{P^2Q^2} \dddot{f}(t) + \dots,$$
$$\dot{\psi} = -\frac{\sqrt{PQ}}{P} \alpha \cos(\sqrt{PQ}\, t + \gamma) + \frac{1}{P} f(t) + \frac{1}{PQ} \dot{g}(t) - \frac{1}{P^2Q} \ddot{f}(t) - \frac{1}{P^2Q^2} \dddot{g}(t) + \dots. \tag{12}$$

Integration of the equations gives the following expressions for the orientation of the figure axis:

$$\theta = \theta_0 - \frac{\alpha}{\sqrt{PQ}} \cos(\sqrt{PQ}\, t + \gamma) - \int \frac{1}{Q} g(t)\, dt + \frac{1}{PQ} f(t) + \frac{1}{PQ^2} \dot{g}(t) - \frac{1}{P^2Q^2} \ddot{f}(t) + \dots,$$
$$\psi = \psi_0 - \frac{\alpha}{P} \sin(\sqrt{PQ}\, t + \gamma) + \int \frac{1}{P} f(t)\, dt + \frac{1}{PQ} g(t) - \frac{1}{P^2Q} \dot{f}(t) - \frac{1}{P^2Q^2} \ddot{g}(t) + \dots. \tag{13}$$

In the right-hand members of these, the first terms are constant and the second terms mean the free nutation. It is easily seen that the third terms correspond to precession and the Poisson terms of nutation if we neglect the small additive terms in P and Q. Then the fourth and following terms must be the so-called Oppolzer terms of nutation. This can be confirmed directly if we calculate these terms and compare them with the analytical values for Oppolzer terms which are found in Kinoshita (1977).

The result is shown in Table 1 for three pairs of terms of nutation which have a greater Oppolzer

Table 1. Comparison between numerical and analytical for some Oppolzer terms.

Term	Period	Poisson term	4th term	5th term	6th term	Oppol. term	Anal. value
(Obliquity)	days	$\Delta\theta_{poi}$	$\frac{f(t)}{PQ}$	$\frac{\dot{g}(t)}{PQ^2}$	$-\frac{\ddot{f}(t)}{P^2Q^2}$	$\Delta\theta_{opp}$	
$\cos\Omega$	-6798.4	+92277	-10.04	+0.00	+0.00	-10.0	-10.0
$\cos 2\Omega$	13.661	+885	+59.08	+4.69	+0.31	+64.1	+64.1
$\cos(2☾-\Omega)$	13.633	+183	+9.95	+0.97	+0.05	+11.0	+11.0
(Longitude)		$\Delta\psi_{poi}$	$\frac{g(t)}{PQ}$	$-\frac{\dot{f}(t)}{P^2Q}$	$-\frac{\ddot{g}(t)}{p^2Q^2}$	$\Delta\psi_{opp}$	
$\sin\Omega$	-6798.4	+172675	-33.91	-0.00	+0.00	-33.9	-33.9
$\sin 2\Omega$	13.661	+2041	+162.10	+10.81	+0.85	+173.8	+173.7
$\sin(2☾-\Omega)$	13.633	+343	+33.54	+1.82	+0.18	+35.5	+35.5

Epoch: 1900.0, Unit: 0.0001″.

term. The third column of the table gives Poisson term for each nutation term. Using it, the 4th, 5th and 6th columns are calculated which correspond to the 4th, 5th and 6th terms in equations (13), respectively. The 7th column is the sum of these three columns, and gives Oppolzer term. The 8th column is the value by Kinoshita. All the values are evaluated for the epoch of 1900.0. The coincidence is satisafctory to 0.00001″.

In the following, we only calculate the third and following terms in equations (13). They correspond exactly to the analytically given nutation of rigid Earth which constitutes the basis of the IAU nutation series. In carrying out the integration of the third terms, we no longer consider the integrands to be functions of t alone but to contain θ and ψ which are not constant.

4. Perturbing function

Since the fourth and following terms in equations (13) can be calculated easily by numerical differentiation of the functions $f(t)$ and $g(t)$, we take up only the third terms:

$$\Delta\theta = -\int \frac{1}{C\omega\sin\theta\ \{1-(A\bar{\dot{\psi}}/C\omega)\cos\theta\}}\ \frac{\partial U}{\partial\psi}\ dt,$$

$$\Delta\psi = +\int \frac{1}{C\omega\sin\theta\ \{1-2(A\bar{\dot{\psi}}/C\omega)\cos\theta\}}\ \frac{\partial U}{\partial\theta}\ dt. \tag{14}$$

These are the same as those found in Woolard (1953) if we neglect the small additive terms in the denominators and they very nearly give precession and Poisson terms of nutation.

U is the perturbing function due to the attractions of the Moon and the Sun. They are separated into two parts caused by the respective bodies, that is,

$$U = U_{☾} + U_{\odot}.$$

The two constituents have the same form:

$$U_B = -\frac{3k^2m(C-A)}{2r^5}\,z^2 = -\frac{3k^2m(C-A)}{2r^3}\sin^2\delta, \tag{15}$$

where the suffix B means ☾ or ☉, k is the Gaussian gravitational constant and m, r, z and δ are respectively the mass, the geocentric distance, the z-coordinate and the declination of the body referred to the equator of the Earth. The units of m and r are the solar mass and the astronomical unit of distance (a.u.), respectively.

In terms of the ecliptic longitude λ and latitude β of the Moon or the Sun, (15) can be expressed as

$$U_B = -\frac{3k^2m(C-A)}{2a^3}\left(\frac{a}{r}\right)^3 \{\cos\theta\sin\beta_0 + \sin\theta\cos\beta_0\sin(\lambda_0-\psi)\}^2, \tag{16}$$

where a is the conventional unit in which r of the Moon or the Sun is expressed. The suffix 0 assigned to λ and β means that they are referred to the ecliptic and mean equinox of J2000.0.

The coordinates of the Moon and the Sun are taken from an abridged trigonometric series for them developed by Kubo (1980). The error of the series is estimated to be 2″ in average and 10″ at maximum. The effect of the error to the result will be discussed in Section 6.

We now discuss on the quantity $k^2m/\omega a^3$ in the coefficients in equations (14), in which ω is the sidereal mean motion of the rotation of the Earth with the value 1299548.204″/day.

In case of the Moon, we take as a the equatorial radius of the Earth a_e. Introduce $a_☾$ defined by

$$a_☾ = \sqrt[3]{\frac{k^2(m_\oplus + m_☾)}{n_☾^{\,2}}} = 0.002571881428 \text{ a.u.}, \tag{17}$$

where $m_\oplus$ and $m_☾$ are the masses of the Earth and the Moon, respectively, and $n_☾$ (= 47434.88963″/day) is the sidereal mean motion of the Moon. Further, we have a relation among a_e, $a_☾$ and the mean distance of the Moon a_0:

$$a_☾ = a_0/F_2 = a_e/3422.448''/F_2 = 60.32291182\, a_e, \tag{18}$$

F_2 being a constant whose value is 0.999093142. Hence,

$$\begin{aligned}\frac{k^2m_☾}{\omega a_e^{\,3}} &= \left(\frac{a_☾}{a_e}\right)^3 \frac{m_☾}{m_\oplus + m_☾} \times \frac{k^2(m_\oplus + m_☾)}{\omega a_☾^{\,3}} \\ &= (60.32291182)^3 \times 0.01215056777 \times \frac{n_☾^{\,2}}{\omega} \\ &= 4617924.822''/\text{day}.\end{aligned} \tag{19}$$

In case of the Sun, we take 1 a.u. as a. Introduce $a_☉$ defined by

$$a_☉ = \sqrt[3]{\frac{k^2(m_☉ + m_\oplus + m_☾)}{n_☉^{\,2}}} = 1.000000036 \text{ a.u.}, \tag{20}$$

where $m_☉$ is the mass of the Sun and $n_☉$(= 3548.192807″/day) is the sidereal mean motion of the Sun. Then,

$$\begin{aligned}\frac{k^2m_☉}{\omega a^3} &= \left(\frac{a_☉}{a}\right)^3 \frac{m_☉}{m_☉ + m_\oplus + m_☾} \times \frac{k^2(m_☉ + m_\oplus + m_☾)}{\omega a_☉^{\,3}} \\ &= (1.000000036)^3 \times 0.9999969596 \times \frac{n^2_☉}{\omega} \\ &= 9.687701648''/\text{day}.\end{aligned} \tag{21}$$

Finally, as the common factor in $U_☾$ and $U_☉$, we adopt

$$\frac{C-A}{C} = 0.0032739935. \tag{22}$$

All the numerical values adopted above are coincident with the IAU (1976) system of astronomical constans and are the same as those used in the analytical theory.

5. Integration

In carrying out the integration of (14), the integrands depend almost only on t and hardly on θ and ψ because the changes of these variables are very small. Therefore the integration is almost a mere calculation of areas rather than usual numerical integration of equations of motion.

The calculation is carried out by the Simpson's formula for definite integral with a step of 2 hours. In doing this, the perturbing force by the Sun is evaluated at 0^h every day and interpolated to every 2 hours, while for the Moon the coordinates are evaluated at 0^h every day and interpolated to every 2 hours and then the force is calculated. Differences up to the fourth order are taken into consideration in the interpolation. The update of the values θ and ψ is made once a day since the rate of their change is very slow. In doing this, the geodesic precession 1.92″/Jc or 0.0000526″/day is compulsively added to ψ.

The initial values adopted in the integration are as follows:

t_i = JD 2446066.5 or Jan. 1, 1985 0^h DT

$(T_i = -0.1499931553)$,

$\theta_i = 23° 26' 21.448'' + 4.849''$,

$$\psi_i = 180° - 5038.7784'' T_i + 1.07259'' T_i^2 + 0.001147'' T_i^3 + 13.715'', \tag{23}$$

where T is measured from J2000.0 in the unit of Julian century. The values of θ_i and ψ_i are chosen so that they coincide with the analytical values within 0.001″, but it should be noticed that adoption of a slightly different value for θ_i or ψ_i only results a constant bias of the same amount to all the values of θ or ψ throughout the period to be integrated.

6. Result and discussion

The integration has been carried out for a period of about 18,000 days. In the following discussion, ϵ is used in place of θ and $180°-\psi$ in the preceding sections is replaced by ψ, according to the conventional notations used in precession and nutation theory.

As mentioned in Section 2, the nutation obtained in the fixed reference frame $\Delta\psi_0$ and $\Delta\epsilon_0$ must be reduced to the ecliptic and mean equinox of date. The formulae for the reduction are

$$\Delta\psi = \Delta\psi_0 + \pi \cos \Pi \cot \epsilon \, \Delta\psi_0 + \frac{\pi \sin \Pi}{\sin^2 \epsilon} \Delta\epsilon_0,$$
$$\Delta\epsilon = \Delta\epsilon_0 - \pi \sin \Pi \, \Delta\psi_0, \tag{24}$$

where

$$\pi = 47.0029'' T - 0.03302'' T^2 + 0.000060'' T^3 \text{ (in radian)},$$
$$\Pi = 5° 07' 25.018'' - 4168.9695'' T + 1.03723'' T^2 + 0.001147'' T^3. \tag{25}$$

We first examine the short periodic terms of nutation. We compare our result with the analytical one for a period of 250 days beginning from JD 2446066.5. In Figure 2 the differences between our values (denoted by N: numerical) and analytical values (denoted by A) for $\Delta\psi$ and $\Delta\epsilon$ are shown. Figure 3 is their power spectra. The constant biases of about 1 milliarcsecond in $\Delta\psi_{N\text{-}A}$ and $\Delta\epsilon_{N\text{-}A}$ are meaningless because of the reason mentioned in Section 5. As far as short periodic region of nutation is concerned, the differences between N and A in $\Delta\psi$ and $\Delta\epsilon$ are reasonable considering the precision of the analytical computation.

When we proceed to precession and long periodic region of nutation, however, we see a fairly different aspect. Figure 4 shows the $\Delta\psi_{N\text{-}A}$ and $\Delta\epsilon_{N\text{-}A}$ for a period from JD 2445106.5 to JD 2462706.5. In the grapfs, one dot is the average for 32 days. Figure 5 is their power spectra.

In $\Delta\psi_{N\text{-}A}$ all the analytical values of precession and nutation have been subtracted from the numerical solution. Therefore it would be a horizontal straight line if both $\Delta\psi_N$ and $\Delta\psi_A$ were

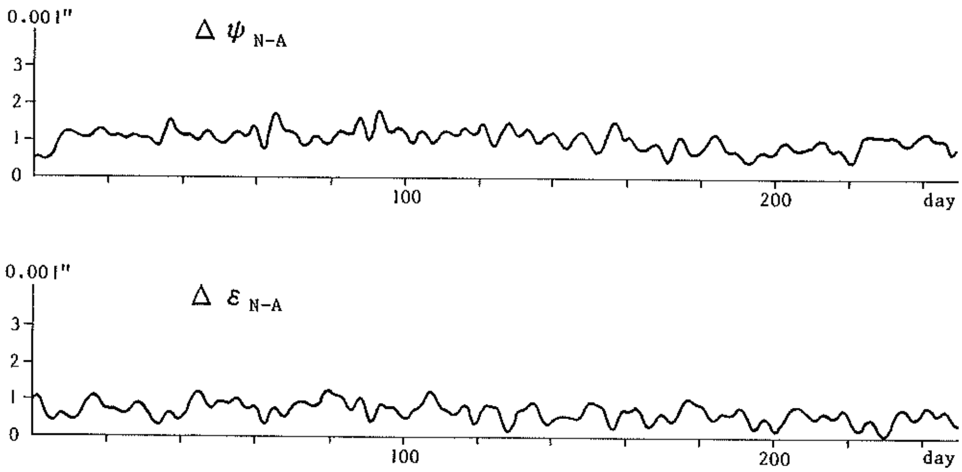

Figure 2. $\Delta\psi_{N\text{-}A}$ and $\Delta\epsilon_{N\text{-}A}$ for JD 2446066.5 to JD 2446315.5.

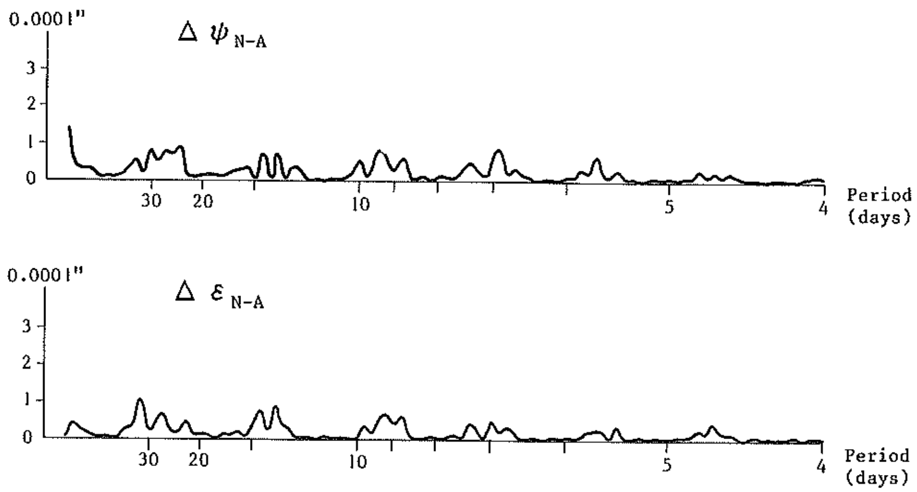

Figure 3. Power spectra of $\Delta\psi_{N\text{-}A}$ and $\Delta\epsilon_{N\text{-}A}$ for short periodic region.

correct. In $\Delta\epsilon_{N\text{-}A}$, however, only the analytical nutation has been subtracted from the numerical result. Therefore from the graph of $\Delta\epsilon_{N\text{-}A}$ should be further subtracted the analytical precession, i.e.,

$$+0.05127'' T^2 - 0.007726'' T^3. \tag{26}$$

An analysis of $\Delta\psi_{N\text{-}A}$ and $\Delta\epsilon_{N\text{-}A}$, where the theoretical precession (26) in $\Delta\epsilon_{N\text{-}A}$ has been removed, gives the following expressions for the differences in precession and long periodic terms of nutation:

$$\Delta\psi_{N\text{-}A} = \underset{(\pm 25)}{+0.0151''} T \underset{(\pm 149)}{- 0.0022''} T^2 + 0.0006'' \sin(\Omega - 26°) + 0.0013'' \sin(2\Omega - 2°),$$

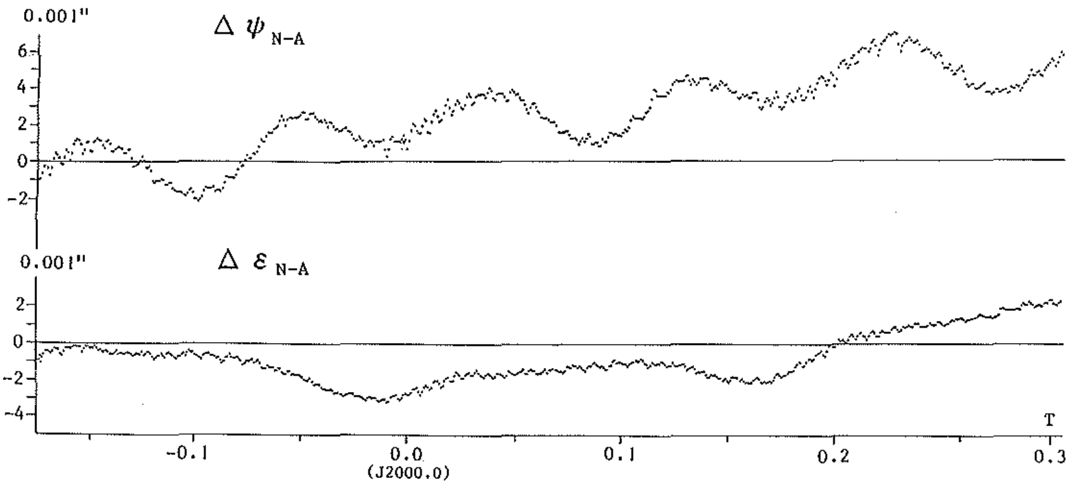

Figure 4. $\Delta\psi_{N\text{-}A}$ and $\Delta\epsilon_{N\text{-}A}$ for JD 2445106.5 to JD 2462706.5.

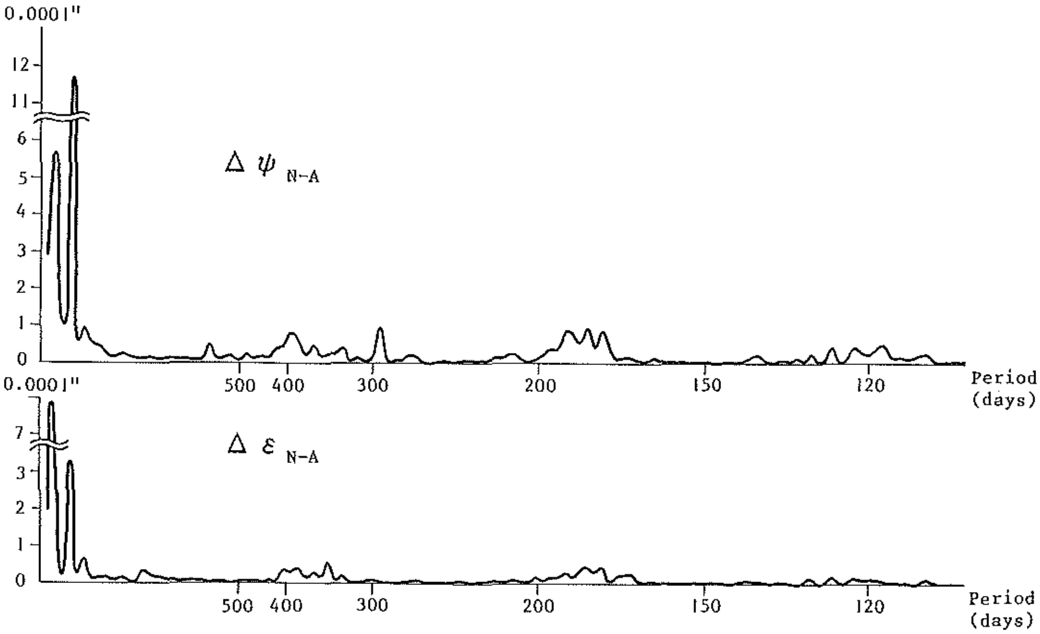

Figure 5. Power spectra of $\Delta\psi_{N\text{-}A}$ and $\Delta\epsilon_{N\text{-}A}$ for long periodic region.

$$\Delta\epsilon_{N\text{-}A} = -0.0003'' T - 0.0067'' T^2 + 0.0008'' \cos(\Omega + 26^\circ) - 0.0003'' \cos(2\Omega + 37^\circ), \quad (27)$$
$$(\pm 12) \qquad (\pm 62)$$

the first and the second terms being for precession and the third and the fourth terms for nutation in each equation. Ω is the longitude of the ascending node of the Moon's orbit on the ecliptic.

Among the four terms for precession in (27), only the linear term $+0.0151'' T$ in $\Delta\psi_{N\text{-}A}$ is significant judging from the mean errors. Since this term is to be determined by observation, the difference is not important physically. However, it must not exist because the same constants are adopted in both the numerical and analytical solutions. All the terms for nutation in (27) are significant. Among them the terms with the argument of 2Ω are well coincident with the result Kubo (1982) obtained analytically:

$$\delta(\Delta\psi) = +0.0012'' \sin 2\Omega,$$
$$\delta(\Delta\epsilon) = -0.0002'' \cos 2\Omega. \quad (28)$$

As for the remaining two terms of nutation $+0.0006'' \sin(\Omega - 26^\circ)$ and $+0.0008'' \cos(\Omega + 26^\circ)$, as well as $+0.0151'' T$ in precession, we can not tell in which side N or A there are errors. The errors arizing from the modification of the equations of motion and from the integration in our solution are estimated to be small enough. The largest source of the error in our calculation would be in the low precision of the coordinates of the Moon and the Sun we have adopted. Especially, some long periodic terms which are missing because of their smallness in the trigonometric series for the Moon might be questionable, although the effect to our result does not seem so large as 0.0002″.

However, it is desirable to follow the present scheme once again, using precise ephemerides of the Moon and the Sun and, if possible, applying a more rigorous formula for numerical calculation.

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歳差・章動の数値積分（要旨）

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天体位置表等の天体暦に掲載されている歳差・章動の数値の基になっている剛体地球の歳差・章動理論は従来，解析的方法によってのみ行われてきた．本稿では，その数値的解法を試み，得られた結果を現行の理論値と比較する．歳差においては有意な差は見られないが，長周期域の章動項に0.001″に達する差異が存在し，これは現行の理論の精度が十分でないためと考えられる．