COMPARISON OF THE LUNAR PROFILES OF WATTS AND OF WEIMER

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Abstract

Direct comparisons are made between profiles of the moon's limb of Weimer and of Watts.

Comparison made at 177 points on the moon's limb shows that Weimer's centre deviates 0″.39 in the position angle of 336° from Watts' centre, while their adopted radii of the moon are practically identical.

Another comparison is made at every 5° along the moon's limb for 9 cases of libration, i. e. combination of $l=-6°$, $0°$, $+6°$ and $b=+6°$, $0°$, $-6°$. Discrepancy between Watts' and Weimer's charts differs for eastern and western limbs from each other as well as for libration. There exists no systematic dependence between relative postions of centres and libration. As a whole, Weimer's chart deviates 0″.43 in the direction of 345° for eastern limb and 0″.27 in the direction of 296° for western limb from Watts'.

The effect of the discrepancy between both charts in position of centre is found clearly in the difference of ΔT evaluated from occultation observations with these respective profile charts, and verifies that our values of Weimer's centre relative to Watts' are adequate.

1. Introduction

At present profiles of the moon's limb by Weimer (1952) and those by Watts (1963) are widely employed for reductions of meridian observation and occultation observation of the moon. Knowledge on the characters of these profiles are indispensable to investigate the motion of the moon.

Hirose (1961) shows systematic difference between these charts at 216 points on the moon's limb at which occultations were observed at the Tokyo Astronomical Observatory from 1950 to 1958.

In the present report comparisons are made between these charts, firstly at occultation points, and secondly at every five degrees along the moon's limb for various librations.

2. Notations and formulae

Let

χ : position angle of a point on the limb measured eastwards from the north point of the moon,

C' : position angle of the axis, and

Π : position angle of the point measured from the central meridian of the moon.

Then, we have

$$\Pi = \chi - C'. \qquad (1)$$

For a given point of Π we read elevation of the limb from mean radius of the moon from each of Watts' and Weimer's charts :

h_w : elevation of the moon's limb taken from the Watts' chart,

h_p : elevation of the moon's limb taken from the Weimer's chart,

and put

$$\delta h = h_w - h_p. \qquad (2)$$

If we impose a relation

$$\delta h = A \cos \Pi + B \sin \Pi + C, \qquad (3)$$

between δh and Π, A and B denote the position of centre of the moon's profile adopted by Watts with respect to that by Weimer, and C denotes the difference of the adopted values of the moon's radius.

3. Comparison at occultation points

This process of comparison is similar to that by Hirose (1961), except that he treats group mean of δh for each range of 45° in position angle. In 1955, 201 occultations were observed visually or photoelectrically at the Hydrographic Office. For the use of reduction Weimer's and Watts' charts have been read for all of the occultations. For the comparison of profiles of these charts, 177 occultations are selected among them so as to make distribution of position angles to be almost equal, at least for the eastern limb.

177 sets of δh and Π are directly applied to (3). Solutions of the method of least squares are

		(p.e.)
$A =$	$-0.''358$	$\pm 0.''041$,
$B =$	$+0.156$	$.031$,
$C =$	$+0.001$	$.026$,

or practically

$$\delta h = +0.''39 \sin(\Pi + 294°).$$

These values nearly agree with those by Hirose (1961), i.e.

		(p.e.)
$A =$	$-0.''480$	$\pm 0.''047$,
$B =$	$+0.103$	$.050$,
$C =$	-0.002	$.042$,

or

$$\delta h = +0.''490 \sin(\Pi + 282°).$$

4. Systematic comparison

We take about 70 points on the moon's limb for every 5° in position angle. Further, 9 cases of libration are selected; they are approximately

(1) $\ell=-6°$, $b=+6°$, (2) $\ell=-6°$, $b=0°$, (3) $\ell=-6°$, $b=-6°$,
(4) $\ell=0°$, $b=+6°$, (5) $\ell=0°$, $b=0°$, (6) $\ell=0°$, $b=-6°$,
(7) $\ell=+6°$, $b=+6°$, (8) $\ell=+6°$, $b=0°$, (9) $\ell=+6°$, $b=-6°$.

In Weimer's charts approximate values of libration are shown at left margin of each profile, while actual values are at right margin. We adopt these librations directly so that Weimer's profiles can be read without interpolation. With graphical interpolation Watts' charts are read for respective librations. Each difference δh in elevation is then obtained.

Results of the method of least squares by (3) are shown in Table 1. In the second column, values of libration are given. The third column shows the range of position angle. In the fourth column numbers of comparison pair are given, since, in some cases, Watts' and/or Weimer's charts drop the limb for the position angles near the poles of the moon. The fifth to seventh columns show solutions through (3).

Finally, all 9 cases of libration are combined together, and solved. Its results are shown in the bottom of Table 1. For convenience' sake, we shall designate these values as A_{oe}, B_{oe} for $0° \leq \Pi \leq 180°$, A_{ow}, B_{ow} for $180° \leq \Pi \leq 360°$, and A_{oc}, B_{oc} for $0° \leq \Pi \leq 360°$. Then

$$A_{oe}/B_{oe}=\tan^{-1}165°, \qquad \sqrt{A_{oe}^2+B_{oe}^2}=0''.43,$$
$$A_{ow}/B_{ow}=\tan^{-1}116°, \qquad \sqrt{A_{ow}^2+B_{ow}^2}=0''.27,$$
$$A_{oc}/B_{oc}=\tan^{-1}172°, \qquad \sqrt{A_{oc}^2+B_{oc}^2}=0''.27.$$

In Fig. 1, centres of the moon's profile of Weimer with respect to those of Watts are shown. White semi-circle without right-half denotes centre for position angle between 0° and 180°, and that without left-half denotes centre for position angle between 180° and 360°. Full circle is for 0° to 360°. Number attatched to each circle is the libration number shown in the first column of Table 1. Combined results for 9 librations are shown by similar symbols with black.

TABLE 1. WATTS' LIMB RELATIVE TO WEIMER'S

No.	Libration ℓ	b	Π	n	A	(p.e.)	B	(p.e.)	C	(p.e.)
	°	°	° °		"	"	"	"	"	"
1	−5.85	+6.04	0∼180	35	−0.27	±0.07	+1.13	±0.17	−0.56	±0.13
	−5.47	+6.67	180∼360	35	−0.45	.06	+0.95	.13	+0.55	.09
			0∼360	70	−0.36	±0.05	+0.33	±0.05	+0.05	±0.04
2	−5.91	−0.23	0∼180	35	−0.12	±0.07	−0.04	±0.07	+0.05	±0.05
	−6.73	+0.67	180∼360	36	−0.13	.06	+0.10	.14	+0.11	.10
			0∼360	71	−0.13	±0.06	−0.03	±0.05	+0.11	±0.04
3	−6.48	−5.84	0∼180	35	−0.44	±0.05	+0.57	±0.12	−0.09	±0.09
	−6.25	−6.38	180∼360	33	+0.26	.07	+0.30	.17	+0.19	.13
			0∼360	68	−0.12	±0.05	+0.25	±0.05	+0.15	±0.04
4	+0.25	+6.48	0∼180	35	−0.35	±0.04	+0.70	±0.10	+0.10	±0.07
	+1.37	+6.50	180∼360	35	−0.22	.06	+0.56	.15	+0.41	.11
			0∼360	70	−0.28	±0.04	+0.43	±0.04	+0.30	±0.03

TABLE 1. WATTS' LIMB RELATIVE TO WEIMER'S (continued)

No.	Libration ℓ	b	Π	n	A	(p.e.)	B	(p.e.)	C	(p.e.)
	°	°	°		"	"	"	"	"	"
5	+1.33	−0.41	0∼180	37	−0.56	±0.06	−0.27	±0.13	+0.24	±0.09
	+1.07	+0.68	180∼360	37	−0.14	.05	+0.60	.12	+0.31	.08
			0∼360	74	−0.35	±0.04	+0.12	±0.05	+0.00	±0.03
6	−0.42	−6.02	0∼180	35	+0.09	±0.08	+0.45	±0.18	−0.39	±0.13
	−0.68	−5.68	180∼360	35	+0.21	.07	+0.64	.17	+0.24	.12
			0∼360	70	+0.16	±0.05	+0.14	±0.05	−0.14	±0.04
7	+5.85	+5.86	0∼180	35	−0.77	±0.05	−0.28	±0.11	+0.15	±0.08
	+6.12	+5.94	180∼360	35	+0.00	.06	−0.54	.13	−0.12	.10
			0∼360	70	−0.38	±0.05	−0.30	±0.05	+0.15	±0.03
8	+6.36	+0.51	0∼180	37	−0.55	±0.08	+0.04	±0.18	−0.18	±0.13
	+5.34	+0.39	180∼360	37	−0.12	.04	−0.15	.09	+0.03	.06
			0∼360	74	−0.34	±0.05	−0.20	±0.05	−0.02	±0.03
9	+6.15	−5.98	0∼180	34	−0.70	±0.06	−0.71	±0.15	+0.40	±0.11
	+5.15	−6.39	180∼360	30	−0.49	.07	−0.70	.17	−0.06	.13
			0∼360	64	−0.60	±0.04	−0.42	±0.04	+0.18	±0.03
1 ∼ 9			0∼180	318	−0.42	±0.03	+0.11	±0.06	+0.02	±0.04
			180∼360	313	−0.12	.02	+0.25	.06	+0.24	.04
			0∼360	631	−0.26	±0.02	+0.04	±0.02	+0.09	±0.01

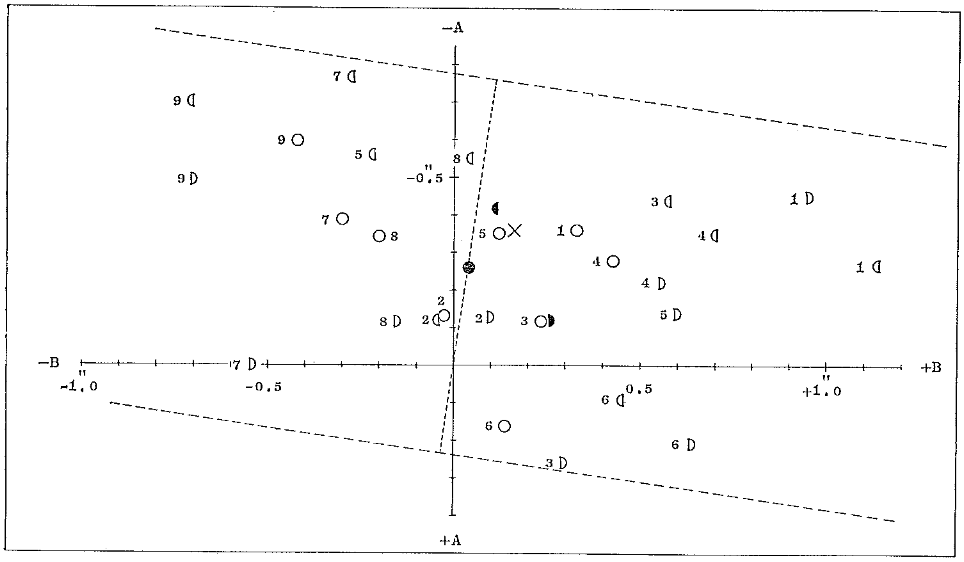

Fig. 1 Locations of Weimer's centres relative to Watts'.

In general, centres of Weimer's limbs lie northward than those of Watts', being distributed in a belt from east-by-north to west-by-south, as is shown by dashed lines in the figure. It is remarkable that these two dashed-lines lie practically parallel from each other and that a perpendicular from the coordinates origin to the dashed lines passes through the centre (A_{oc}, B_{oc}). For librations of $\ell=+6°$ (cases 7, 8 and 9) Weimer's centres are situated eastward than Watts'; for $\ell=-6°$ (cases 1, 2 and 3) contrary tendency is not clear. As a whole, Weimer's centre of eastern limb lie about $0''.43$ apart in the direction of 345° from that of Watts, with almost identical radius. Weimer's centre of western limb lie about $0''.27$ apart in the direction of 296° from that of Watts, with radius $0''.24$ smaller than Watts'.

The result from occultation points obtained in the last section is shown by a cross in Fig. 1. These occultations consist of 133 disappearances and 44 reappearances Then, we make mean value of A_{oe} and A_{ow} and that of B_{oe} and B_{ow} weighted by numbers of disappearance and reappearance, respectively. We obtain

$$\overline{A_{oe}+A_{ow}}=-0''.34, \quad \text{and} \quad \overline{B_{oe}+B_{ow}}=+0''.15.$$

These values agree well with those obtained in Section 3, i.e. $A=-0''.36$, $B=+0''.16$. Hence, values of (A_{oe}, B_{oe}) and (A_{ow}, B_{ow}) seem to be practicable to statistical purpose, irrespective to libration.

On seeing values of C in Table 1, we can not find any clear relation between the radius difference and libration. In some case, C for whole position angles becomes much smaller than C's for both ranges of position angle 0° to 180° and 180° to 360°.

5. Effect to ΔT-evaluation

When we employ Weimer's charts to correct the moon's limb for reduction of occultation observation, resulted value of ΔT may be different from that through Watts' charts. This difference may be approximated by $\delta(\Delta T)=1^s.82\cdot B$.

For the value of $B=+0''.16$ obtained in Section 3, as an example, the expected difference of ΔT is $\delta(\Delta T)=+0^s.29$. In fact, values of ΔT from occultation observations of 201 stars in 1955 are

	(p.e.)	
$\Delta T_p=+30^s.04$	$\pm 0^s.08$	with Weimer,
$\Delta T_w=+29.75$	.03	with Watts,

in Brouwer's system. Then, $\Delta T_p-\Delta T_w=+0^s.29$. The agreement seems to plausibly good.

Another example can be found in the results of photoelectric observations of occultations at the Tokyo Astronomical Observatory. Aoki (1961) gives values of ΔT with Weimer and Watts separately, for 1952.5 to 1958.5. Authors have incidentally evaluated ΔT's for 1960.5 to 1962.5, except for the observations made at temporary stations of the observatory. Table 2 gives the differences of these ΔT's. On the other hand, for the values of B_{oe} and B_{ow} in Table 1, $\delta(\Delta T)$'s are $+0^s.21$ for $0°\leq\Pi\leq 180°$ and $+0^s.45$ for $180°\leq\Pi\leq 360°$, respectively. Thus, we can find that the range of $\Delta T_p-\Delta T_w$ in Table 2, except for the years of 1958 and 1961, agrees satisfactorily with above values of $1^s.82\cdot B_{oe}$ and $1^s.82\cdot B_{ow}$, though the dependence on the distri-

TABLE 2. DIFFERENCE IN ΔT- EVALUATION

	$\Delta T_p - \Delta T_w$	Distribution of Position Angle	
		$0° \leq \Pi \leq 180°$	$180° \leq \Pi \leq 360°$
1952.5	+ 0.26 s	20	0
53.5	+ .36	13	0
54.5	+ .19	48	3
55.5	+ .24	33	1
56.5	+ .30	32	3
57.5	+ .47	21	1
58.5	− .07	19	5
1960.5	+ 0.40	18	4
16.5	+ .11	28	4
62.5	+ .46	18	5

bution of Π does not appear in this case.

6. Conclusion

Discrepancy between the profiles of the moon's limb by Watts and those by Weimer differs for eastern and western limbs from each other as well as for libration. As a whole, Weimer's centre is situated at a point $0''.27$ apart in direction of 352° with radius $0''.09$ smaller than Watts'. Positions of Weimer's centres relative to Watts' for eastern and western limbs obtained respectively in the present paper seem to be practicable for statistical researches.

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(Astronomical section)

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