水路部研究報告　第32号　平成8年3月27日
REPORT OF HYDROGRAPHIC RESEARCHES No.32　March, 1996

Interpolation of Bathymetric data using two-dimensional DFT approximate expression †

Akira ASADA*

Hydrographic Department of Japan †

Abstract

For establishment of the more advanced interpolation method of bathymetric mesh data, the interpolation method using two-dimensional Discreet Fourier Transform approximation of 32* 32 or 64* 64 mesh data has been studied. This interpolation method has high capabilities of topographical approximation. Since it is able to grasp the lineament and features of complicated topography well using many coefficients, it has high ability of the interpolation. The basic process to obtain two-dimensional Discreet Fourier Transform approximation is the repeated calculation of each frequency components by least squares method. Especially, it is important to repeatedly calculate the small frequency components which have longer wave length than the area of deficit part. The calculation order among directional components of same frequency follows their approximate values. In according to the deficit condition of bathymetric mesh data, frequency components of calculation are selected in advance. For the area of large deficit part, trial results of this method were satisfied.

1. Introduction

Several problems exist when a seafloor topographical map is prepared from multi-beam data. We must solve these problems. One of them is interpolation of deficit parts. The approximate interpolation program using linear, quadratic, and cubic polynomial expressions has been established and has been used until now (Asada et al, 1989). As a result, a good result was obtained in the case of small blank area. However, there were various problems for interpolation of large deficit parts of data. In a large deficit part case, it is possible to create undesirable artificial topography. However, personnel in correction of the computer drawing contour chart could not always notice the undesirable artificial topography.

In order to solve large deficit part case, the author paid attention to two-dimensional Discreet Fourier Transfer (DFT), one of the techniques used to analyze seafloor topography. He studied the two-dimensional DFT approximation expression that can effectively process features of seafloor topography, and completed a practical interpolating method using that expression.

2. The cubic polynomial approximate interpolation

One purpose for this study is aimed at advanced interpolation method of seafloor topographical data. The cubic polynomial approximation method of X-Y with 16 coefficients has

† Received 1996 February 1st　Accepted 1996 March 7th
* 海洋研究所　Ocean Research Laboratory

been used up to now.

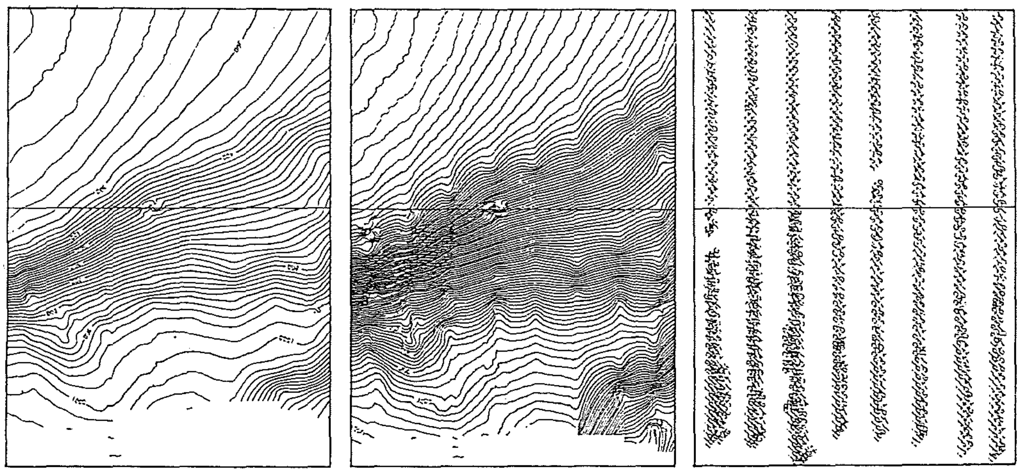
$$Z=\sum_{m=0}^{3}\sum_{n=0}^{3}(a_{mn}*X^{m}*Y^{n}) \qquad (1)$$

In the case of small deficit parts, they can be interpolated successfully using the cubic polynomial expression above. In Fig. 1, a sounding chart (3) shows the distribution of mesh depth data with blank rows equivalent to about 5 times as a mesh size. In case of this scale deficit parts, the interpolation using the plane approximation gave us a unsatisfactory result (2). But, we could get a satisfactory contour chart (1) by the interpolation using the cubic polynomial approximation. Only in case of interpolation using the cubic polynomial approximation, various problems for larger deficit parts of data occur. In a large deficit part case, the sampling size must be expanded for the interpolation process. However, the features of large-scale topography cannot be grasped correctly using an approximate expression only with 16 coefficients. The problem of divergence occurs also when the order of polynomial expression increases.

The whole sampling block cannot be approximated well in this case. Since the topography of a cliff, valley, and fault cannot be approximated well, it approximates roughly and requires a matching process with the real data.

For the flat region under a cliff, a fine flat plane is naturally created by sediment in the extension of the cliff. If, however, a large deficit part of data exists around these places, artificial topography such as landslip, hole, and knoll were frequently created by the interpolation (see Fig. 3). Editing personnel could not always notice incorrect portions of the interpolated seafloor topography map. Fig. 2 shows this example of contour chart with undesirable artificial topography.

In the conventional processing system, the topography map is interpolated using a computer, and each defective region has been manually modified by the engineer. Therefore, it is necessary to establish the more advanced interpolation method and a program not dependent upon modification by the editors.



(1) contour interval 20m (2) contour interval 10m (3) sounding chart

Fig. 1 Mesh data interpolation using the cubic polynomial approximation

(1) Result of satisfactory interpolation using the cubic polynomial approximation

(2) Result of unsatisfactory interpolation using the plane approximation

(3) Distribution of mesh depth data with blank rows equivalent to about 5 times as a mesh size

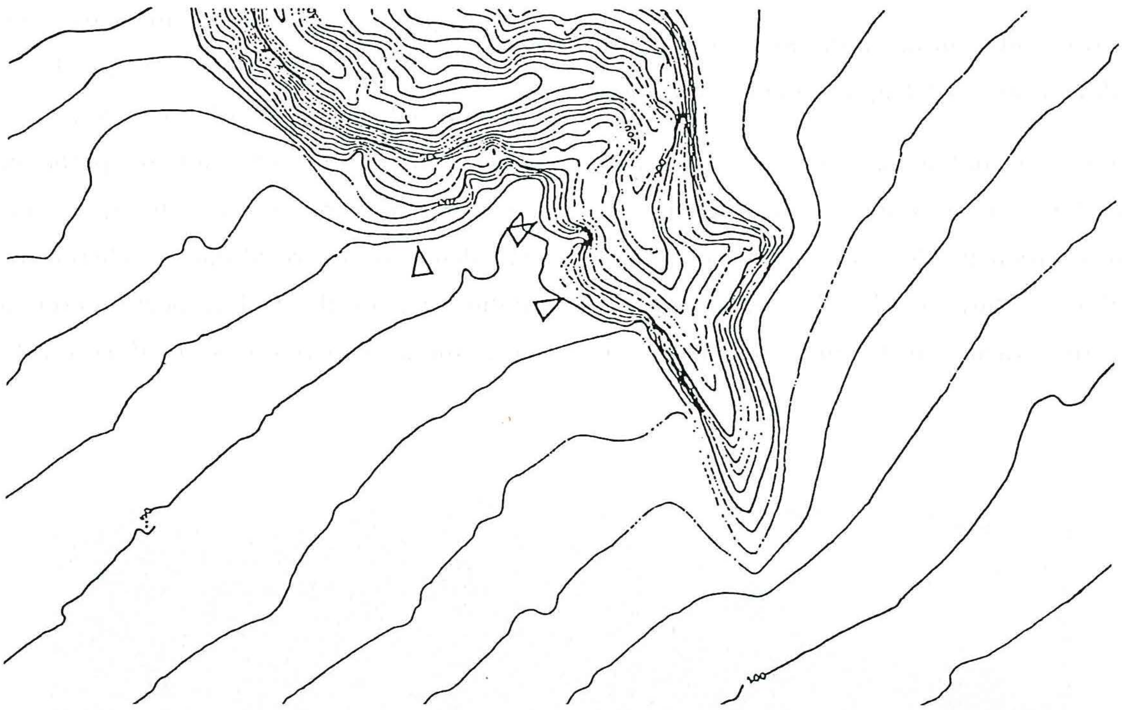


Fig. 2 Contour chart with undesirable artificial topography (marked with Δ) was processed by the cubic polynomial approximate interpolation

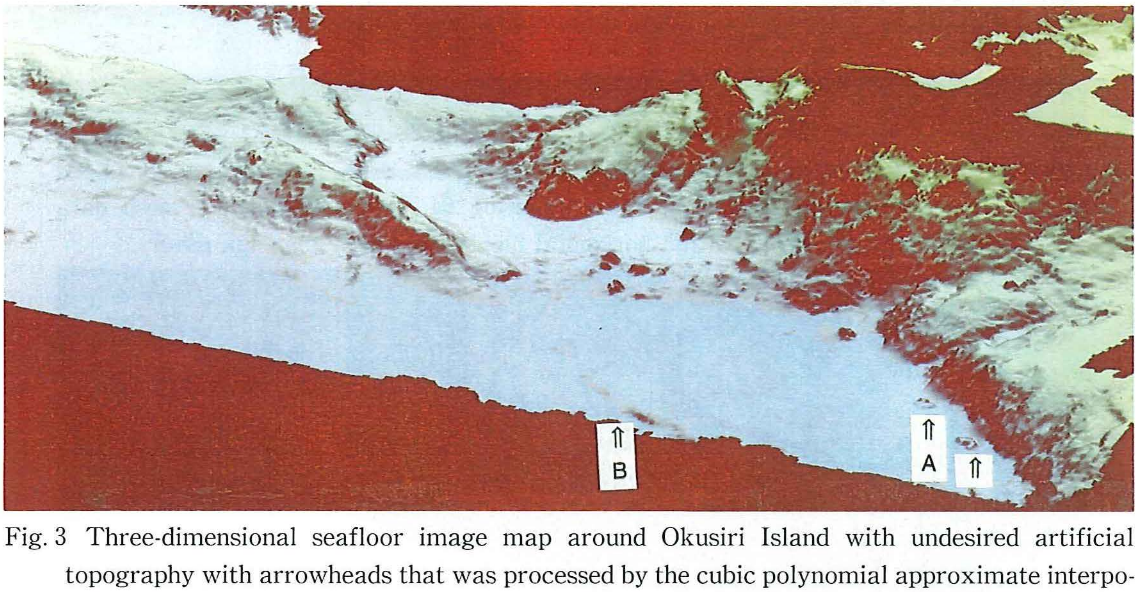


Fig. 3 Three-dimensional seafloor image map around Okusiri Island with undesired artificial topography with arrowheads that was processed by the cubic polynomial approximate interpolation

3. Interpolation of mesh data using two-dimensional DFT approximate expression

First, the author sets his eyes on two-dimensional Fast Fourier Transform with many coefficients. For interpolation of large deficit parts showed in Fig. 4 , it is nessesary to correctly grasp the features of large-scale topography. Several measurement topographical data had been processed by the two-dimensional FFT of 64*64. However, the two-dimensional FFT could not grasp the lineament of topography completely. Such a defect was detected in several places. Therefore, the author has established a new interpolation program using two-dimensional Discreet FT

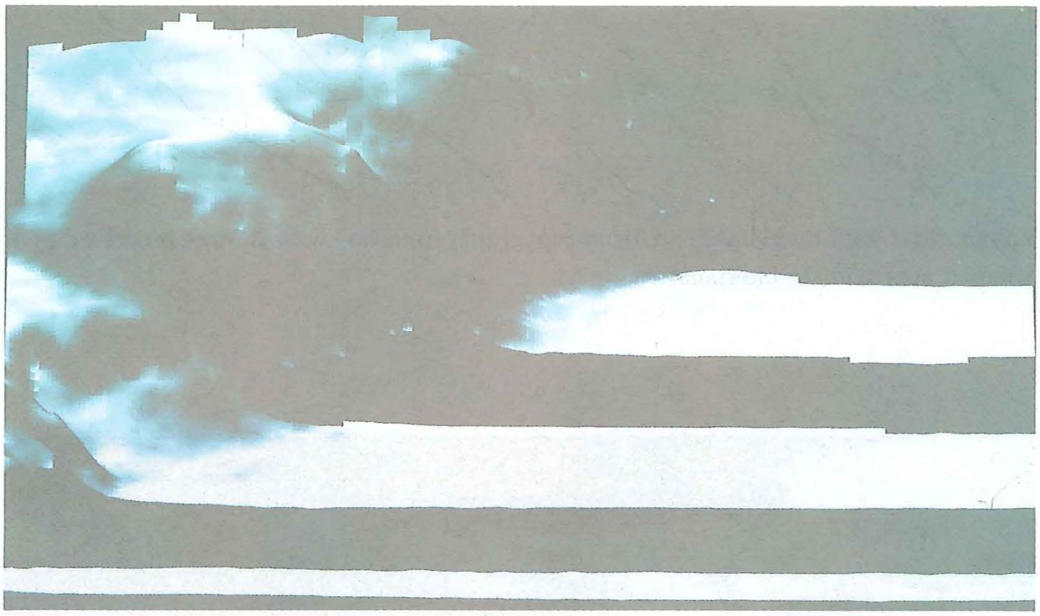


Fig. 4 Three-dimensional seafloor image map using 64 x 64 measurement mesh data around A point in Fig. 3 with a horizontal blank row larger than 12x mesh

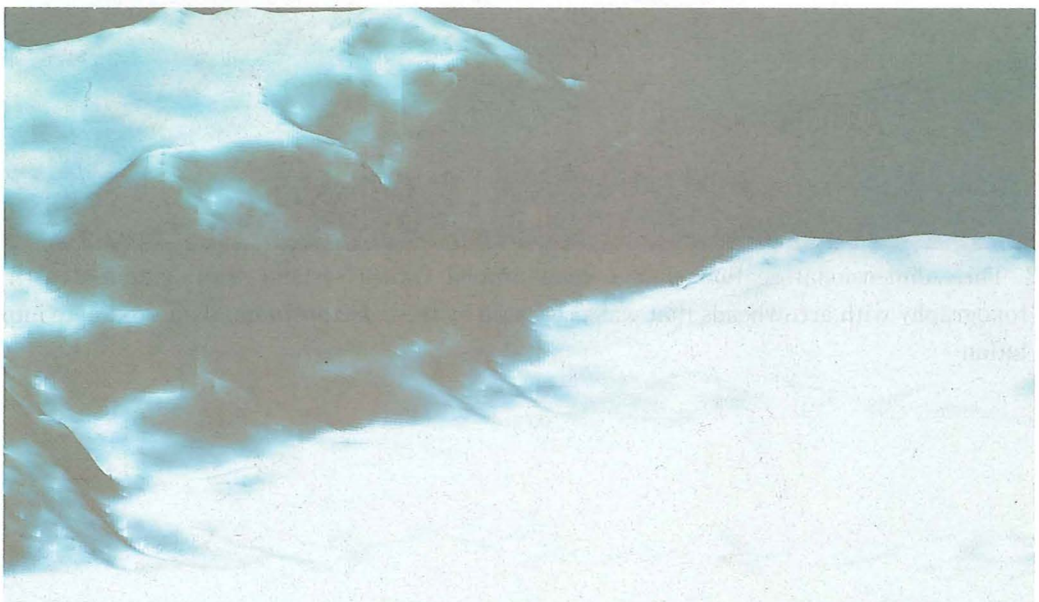


Fig. 5 Three-dimensional seafloor image map by two-dimensional DFT approximation of the area in Fig. 4 with the 64 x 64 mesh size

approximate method that compensates for this defect. The interpolation using the two-dimensional DFT approximation give us a satisfactory result in Fig. 5.

The two-dimensional DFT is given by the following expression.

$$Z' = \sum_{m=0}^{32} \sum_{n=-32}^{32} \{ a_{mn} * \cos(\omega_m X + \omega_n Y) + b_{mn} * \sin(\omega_m X + \omega_n Y) \} \quad (2)$$

To grasp the lineament and features of complicated topography well, each component of $(\omega_m X + \omega_n Y)$ is calculated in the order of increasing frequency and catching precise characteristics of topography. At each step, the component value is subtracted. The ω_m contains only a positive region, and the ω_n not only a positive component but also a negative region.

In this approximate method, each frequency component is piled up with deficit parts of data. In other words, some characteristics of topographies with different liniations are piled up. To approximate to sample data, approximate calculation is performed irrespective of the blank area in data. Therefore, the maximum amplitude of real data is monitored in the process of approximation. The data with a frequency component higher than the maximum amplitude of real data in this stage is eliminated as an error.

The reference frequency with wavelength of about 1.5 to 2 times as high as the maximum sampling size of mesh data, is selected so that each frequency is the integer multiple of its reference frequency. As a result, the whole topography of the mesh data could be approximated better. The parameter for this wavelength is used to correspond to the mesh size and the topography scale of the area.

Sequential calculation from a low frequency component is recommended. Moreover, the topography can be grasped well by repeating the approximate calculations of low frequency components. The greatest problem of the two-dimensional DFT is that a frequency component in an X-Y direction can be also approximated using the other frequency component with different or same wavelength in other X-Y direction. In other words, a program must be designed so that it is best matched for the seafloor topography.

An unnatural approximation occurs in the deficit parts of data due to the relation between the deficit distribution status of data and the direction of a frequency component. Therefore, a filtering function was added for the frequency component in the direction easily influenced by deficit distribution of data, and is not calculated.

In the cubic polynomial approximate expression of X-Y, the Z value significantly diverges when position is far from (0, 0). However, there is no divergence in the two-dimensional DFT. The number of coefficients basically becomes 4096 (32*64*2) and can grasp the topography accurately. However, considerable calculation time is required. It takes five minutes to obtain a satisfactorily approximate expression of 64*64 using the program in the current stage. A work station of arithmetic speed 100MIPs is used in this case. Since some ten thousands of interpolation points must be calculated practically, in the current stage, the approximate size is prescribed as units of 32*32 due to limitation in the arithmetic operation speed of a computer. It is necessary to use the units of 64*64 if the deficit region becomes very large.

The interpolation capability was improved by limiting each frequency component calculated according to the data deficit status. Filtering was designed before approximate calculation. For an example given in Fig. 7 , the frequency component in the X direction is limited in a range 0 to 12 times as high as the reference least frequency. The frequency component in the Y direction is limited in a range 0 to 3 times as high. In this case, a interpolation result was highly improved. About ten days were required for calculation to interpolate all the deficit regions in Fig. 7 . The result using the two-dimensional DFT approximation in Fig. 7 was improved—more so than unsatisfactory result of the approximate methods using the cubic polynomial expression of X-Y in Fig. 6 .

4. How to obtain two-dimensional DFT approximate expression

(Basic expression and it's definition)

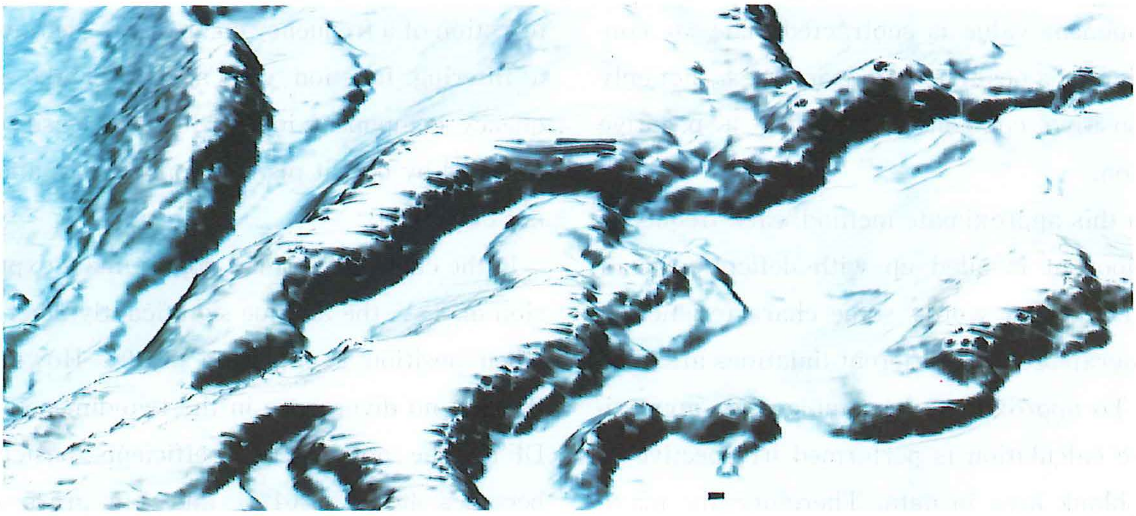


Fig. 6 Unsatisfactory three-dimensional seafloor image off Akita by linear-cubic polynomial approximate interpolation in the 520 x 460 mesh size. It took about 10 minutes to complete processing.

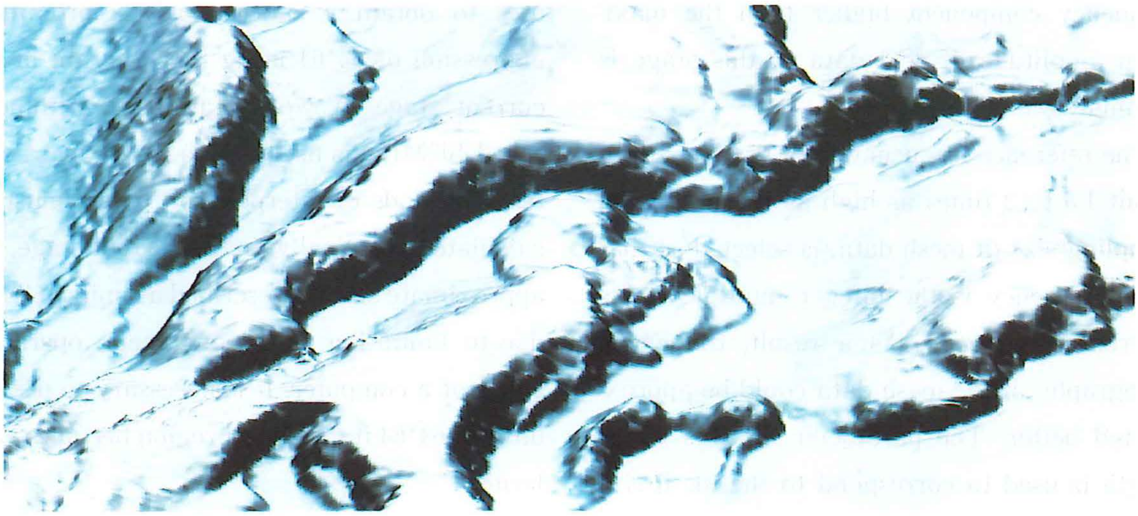


Fig. 7 Improved three-dimensional seafloor image off Akita by two-dimensional DFT approximate, interpolation in the 520 x 460 mesh size. It took about seven days to complete processing.

$$Z' = \sum_{m=0}^{32} \sum_{n=-32}^{32} \{a_{mn} * \cos(\omega_m X + \omega_n Y) + b_{mn} * \sin(\omega_m X + \omega_n Y)\} \quad (3)$$

ω_m : Frequency component that changes with an X component. m is the filter size of 0 to K_x.

ω_n : Frequency component that changes with a Y component. n is the filter size of $-K_y$ to K_y.
The fixed magnification (about 1.0 to 3.0) that preset the ordinary DFT components (ω_m and ω_n) determined by N is multiplied. $0 \leq K_x \leq N/2$, $0 \leq K_y \leq N/2$

a_{mn}, b_{mn} : Indicates the amplitude and phase of m and n frequency components.

N : Input data size $0 \leq X \leq N$, $0 \leq Y \leq N$

$Z(X_i, Y_i)$: Input sample data i=1 to M

(Calculation order)

For K=0, obtain a_{mn} and b_{mn}.
For K=1, obtain a_{mn} and b_{mn}.
For K=2, obtain a_{mn} and b_{mn}.
....

Calculation in steps a) and b) is performed in the combination of $(m=K, -K \leq n \leq K)$ and $(n=\pm K,\ 0 \leq m \leq K)$ under the condition of $(-K_x \leq m \leq K_x)$ and $(-K_y \leq n \leq K_y)$.

a) a_{mn} and b_{mn} are calculated in a pair at a time by least squares method. In this process, Z(X, Y) does not change.

The square sum d(m, n) is calculated by summing difference between of Z(X, Y) and the value obtained from a_{mn} and b_{mn} components. In the position where no data is contained, this value is not used for calculation as $Z(X_i, Y_i)=0$.

The square sum of the difference between the measurement data and a_{mn} and b_{mn} components is given by the following expression.

$$\epsilon^2 = \sum_{i=1}^{M} \sum_{m=0}^{kx} \sum_{n=-ky}^{ky} \{a_{mn} * \cos(\omega_m X_i + \omega_n Y_i) + b_{mn} * \sin(\omega_m X_i + \omega_n Y_i) - Z(X_i, Y_i)\}^2 \quad (4)$$

Square sum ϵ^2 becames least under the following conditions.

$$\delta \epsilon^2 / \delta a_{mn} = 0$$
$$\delta \epsilon^2 / \delta b_{mn} = 0 \quad (5)$$

As a result, a_{mn} and b_{mn} components can be obtained from the expressions below.

$$a_{mn} * \Sigma\Sigma\Sigma\cos^2(\omega_m X_i + \omega_n Y_i) + b_{mn} * \Sigma\Sigma\Sigma\cos(\omega_m X_i + \omega_n Y_i)\sin(\omega_m X_i + \omega_n Y_i) = \Sigma\Sigma\Sigma Z(Xi, Yi)\cos(\omega_m X_i + \omega_n Y_i) \quad (6)$$

$$a_{mn} * \Sigma\Sigma\Sigma\cos(\omega_m X_i + \omega_n Y_i)\sin(\omega_m X_i + \omega_n Y_i) + b_{mn} * \Sigma\Sigma\Sigma\sin^2(\omega_m X_i + \omega_n Y_i) = \Sigma\Sigma\Sigma Z(X_i, Y_i)\sin(\omega_m X_i + \omega_n Y_i) \quad (7)$$

b) a_{mn} and b_{mn} are re-calculated in the ascending order of d(m, n) by least squares method. At the same time, $Z(X_i, Y_i)$ is subtracted by the component of a_{mn} and b_{mn}. The maximum amplitude among the latest $Z(X_i, Y_i)$ is found out after each step of calculating a pair of a_{mn} and b_{mn}. In the above process, the a_{mn} and b_{mn} data with a component higher than this maximum amplitude is disregarded as an error.

....
....

For $K=\max(K_x, K_y)$, obtain a_{mn} and b_{mn}.

The basic process is as described above. For small K components, the repeated calculation is executed properly. At that time, new values are added to the latest a_{mn} and b_{mn} values.

A region of n is generally 0 to N/2, but in this system, it is -N/2 to N/2. The reason is that when a region of n is 0 to N/2, the direction of frequency component is limited only a region of 0 to 90degrees or 180 to 270 degrees in a cylindrical coordinate system. For $(\omega_m X + \omega_n Y)$ and $(\omega_m X - \omega_n Y)$, different properties are given. For $(-\omega_m X + \omega_n Y)$ and $(\omega_m X - \omega_n Y)$, the signs of a_{mn} and b_{mn} are only inverted, and the same topographical property is obtained.

At last, approximated Z'(X, Y) is obtained by inversion of two-dimensional DFT from determined a_{mn} and b_{mn}. For only the center value of approximate Z'(X, Y), the approximate value is directly used. In this case, the center position of Z'(X, Y) is matched to the interpolation point. In another case, the center value is modified by the matched process between Z'(X, Y) in inverse proportion of square distance from the interpolation point.

5. Conclusion

Interpolation using the two-dimensional DFT approximation requires a large amount of processing time. However, this interpolation is expected to be effective in improving the level of subsequent editing and in obtaining high quality results. How to determine the parameters corresponding to seafloor topography, and how to approximate with excellence, must be continuously studied in future.

The two-dimensional DFT approximation method is able to be directly used for the frequency analysis of seafloor topography. Other applications being considered include the texture analysis of topography, the elimination of noise in topography, the metamorphism of topography, and the analysis of activities. This method covers for the faults in two-dimensional FFT. Therefore, it enables an advanced analysis.

The author thanks Professor Asahiko Taira of Ocean Research Institute, Tokyo University and Professor Tomoyoshi Takeuchi of the University of Electronic-Communication for their valuable instructions and advice provided for the author in preparing this paper.

Reference

A. Asada : 3-D image processing of Sea Beam bathymetric data-as applied from the Sagami trough to the Izu-Ogasawara trench, *Report of Hydrographic Researches,* **21**, 113-133, (1986).

A. Asada and A. Nakanishi : Contour processing of Sea Beam bathymetric data, *Report of Hydrographic Researches,* **21**, 89-110, (1986).

A. Asada and S. Kato and S. Kasuga : Tectonic landform and geological structure survey in the Toyama Trough, *Report of Hydrographic Researches,* **25**, 93-122, (1989).

2次元DFT近似式を使った海底地形データの補間法(要旨)

浅田　昭

海底地形データ処理過程において、海底の地形の特徴を上手く捉えた優れた水深メッシュデータの補間法の確立が望まれている。このため、64×64または32×32個の2次元DFT近似式を使った補間処理法の研究を行い、かなり満足のいく補間法を作成した。2次元DFT近似式は非常に沢山の係数を持っており、複雑な海底地形を表すことや、地形のリニアメント等の特徴を捉えるができる。それ故、高い補間能力を持っている。基本的な2次元DFT近似式の求め方としては、地形の特徴をうまくとらえることが重要である。地形データから最小自乗法を使い、繰り返し計算により最良の各周波数成分の近似値に近づける方法を取った。得に、テータの欠損部の大きさよりも長い波長を持つ低周波領域において、最初に、入念な繰り返し計算により近似値を求めることが肝心である。最大波長もDFT計算データ領域より1.5倍程度大きくすると良い結果が得られる。また、計算順序についても、低周波成分の方から処理し、同じ周波数でも成分の近似値の大きい方向のもの

から順に計算し、その成分値を除去しながら繰り返し計算を行う等の工夫を施した。データの欠損状況に応じて、求める周波数成分を予め制限するフイルター機能も付加した。こうして得られた2次元DFT近似式を使って、かなり大きなデータの欠損部を持つデータについて良い補間結果が得られた。